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Math misconceptions: Mistakes, misunderstanding, and confusion

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ABSTRACT

In this study, we investigate mathematical misconceptions, particularly prevalent among high school students, and offer solutions. The primary aims of our research include identifying the most common mathematical misconceptions, uncovering their underlying causes, and evaluating research-based solutions that have successfully addressed these issues. For this study, data were collected using a mixed-methods approach in Stirling Schools operating throughout Iraq. Data were gathered via a diagnostic test, as well as teacher interviews and classroom observations. The test revealed conceptual, procedural, and application problems, while interviews and observations gave qualitative information about the sources of misunderstandings and teaching tactics. The data analysis blended quantitative mistake classification with qualitative theme analysis to provide a complete picture of misunderstandings and effective training approaches. In light of the data obtained, we can state that misconceptions typically stem from a lack of conceptual understanding, over-reliance on procedural methods, and contextual misunderstandings. Additionally, it has been shown that conceptual education, visual aids, and peer-supported learning help eliminate mathematical misconceptions. These valuable findings hold significant implications for educators seeking to improve students' mathematical knowledge by emphasizing the importance of concept-based learning and providing opportunities for re-education. Through this study, we offer key insights into how widespread mathematical misconceptions can be effectively addressed, contributing meaningfully to mathematics education.

INTRODUCTION

Students should be taught mathematics at an early age to foster creative thinking, logic, and hard effort. However, many students still consider mathematics is difficult (Yusup, 2023). According to Irawan & Wilujeng (2020), teaching mathematics seeks to increase students' thinking abilities and cognition, change their behavior for the better, and allow them to create works based on mathematical concepts. Mathematics is a scientific field that studies facts, concepts, processes, and abstract ideas. Fujii (2020) describes research goals for mathematics education as facts, concepts, principles, and abilities. Each idea and structure is organized in a methodical fashion, so they always go from the most fundamental to the most complicated concepts and structures. Students will learn advanced topics more effectively if they have a firm understanding of the fundamental concepts. If students do not understand earlier preconceptions, it may cause misunderstandings in future working (Radiusman, 2020).

Several factors contribute to students' learning difficulties and lack of conceptual comprehension, including the availability of instructional materials and the learning approach used (Chew & Cerbin, 2021). Students typically struggle with "getting" new concepts when they first encounter them. Achieving conceptual growth and mastery may take hours, days, weeks, months, or years. Students make frequent mathematical errors, which we call misconceptions, but a more sympathetic and forgiving phrase is 'fresh beginnings.' Paying close attention to frequent misconceptions and incorporating them into instructional sections is a powerful teaching method.

There are various components to student thinking. Their mathematical attitudes and thoughts include formulas, relevance, tedium, and enjoyment. Misconceptions caused by inadequate teaching, informal review, or poor remembering are one source of major learning difficulties in mathematics. We must recognize the need to strengthen schools to design a more productive future and the critical role of education in preparing the developing generation to live at the desired standard (Kurudirek & Berdieva, 2024).

Misconceptions in mathematics are recurrent issues for both students and teachers. "Misconceptions" are also referred to by synonyms such as incorrect ideas, preconceptions, alternate viewpoints, naive beliefs, and prescientific notions. A misconception is a part of a mistake, not the other way around, meaning a mistake can stem from a misunderstanding, and a misconception can lead to errors. A misconception differs from an alternative thought; the former contradicts scientific views, while the latter is a learner's idea rejected by scientists but not necessarily incorrect (Al-Osaimi, 2019). Additionally, as noted by diSessa (2014) and Özdemir & Clark (2007), students' prior knowledge can obstruct learning and hinder their understanding of scientific concepts.

Alternative notions and misconceptions are often difficult to change because individuals tend to defend them as they align with their valued conceptual structures and beliefs. A misperception is a false mental image formed due to confusing or inadequate educational experiences, leading to operational errors in problem-solving. However, not all operational errors result from alternative understanding; sometimes, they arise from missing steps or misplacing values (Alkramiti, 2023). Some researchers prefer using "alternative conceptions" instead of misconceptions to avoid negative connotations, though the terms are often considered interchangeable (Clement, 1993; Taber, 2015).

There are six types of misconceptions students may face in mathematics (Rahmawati et al., 2021):

1. Translation misconceptions, which are faults in interpreting the conversion of information into mathematical expressions or defects in understanding the conversion of mathematical problems into correct words.
2. Misconceptions about signs, namely writing errors, as well as operations or symbols.
3. Misconceptions about concepts, notably errors in applying and connecting one concept to another, or flaws in presenting statements that contradict previously taught scientific principles.
4. Strategic misunderstandings are errors that occur when students pick the wrong path, which leads to a dead end and makes it difficult for them to solve the problem.
5. Systematic misunderstandings are errors in the choice of solution steps.
6. Misconceptions about calculation, including mathematical computation errors.

Despite substantial research on mathematical misconceptions, there is a lack of a holistic understanding that considers both conceptual errors and contextual issues. Furthermore, most research has used just qualitative or quantitative approaches, not both, and has disregarded real-time classroom dynamics and instructor interventions for resolving misunderstandings.

This study fills the gap by employing a mixed-methods approach that includes diagnostic tests, teacher interviews, and classroom observations to give quantitative and qualitative insights on misunderstandings. The study also focuses on how teachers handle misunderstandings in real time and offers concrete solutions for resolving these misconceptions using specialized teaching strategies. Numerous studies have shown that students develop certain mathematical misconceptions due to factors such as reliance on rote memorization, overconfidence in their ability to recall information, cognitive overload during problem-solving, and the inability to apply previously taught concepts appropriately.

Among this web of problems, this study aims to both identify the roots of these beliefs, particularly among high school students, and evaluate practical solution methods to help them overcome these challenges, focusing on the effectiveness of research-based interventions.

Furthermore, this research examines data from similar studies and proposes approaches that can enhance students' mathematical knowledge. Misconceptions, if not corrected early, can collect over time and hamper the learning process, as mathematics is based on sequential knowledge.

Why Research Misconceptions

Our initial idea is that once imprinted in a student's memory, misconceptions are tough to overcome. Numerous studies (Confrey, 1990; Osborne & Wittrock, 1983) supporting this have aroused academics' interest in student views.

1. Prior to the formal study, individuals had established strong descriptive and explanatory frameworks for scientific and logico-mathematical phenomena, i.e., belief systems about mathematics.
2. These belief systems differ from those included in the regular curriculum.
3. Certain combinations of these belief systems are extremely consistent across ages, talents, and cultures.
4. Traditional training is resistant to altering belief structures.

Students are emotionally and intellectually engaged in their misunderstandings, which they actively develop and, in certain cases, give easy solutions to various difficulties. When students use them to analyze novel situations, they impede learning. Recognizing student misunderstandings and re- educating them to think mathematically is crucial.

Several studies have also shown links between math anxiety misconceptions and the impact of accepting math anxiety myths, mathematical self-concept, and arithmetic skills on math anxiety. Misconceptions, mathematical self-concept, and arithmetic skills were identified as major determinants of statistics course achievement. Negative views toward mathematics delayed older students' return to school after an extended break. It was established that mathematics anxiety is defined by a mechanical, nonconceptual approach to arithmetic, a lack of confidence, and a tendency to give up easily when answers are not immediately evident.

Literature Review

An increasing amount of research investigates the frequency of mathematical misunderstandings and their effects on student learning. Vinner (1997) established the idea of "pseudo- conceptual understanding," which emphasizes the difference between students' perceived and real knowledge. According to his results, students may appear to be skilled at solving mathematical problems, but their blunders indicate a lack of a thorough comprehension of fundamental ideas. For example, students may comprehend how to use the distributive property mechanically yet misapply it when variables or more sophisticated expressions are presented.

Exacta et al. (2024), who examined students' mathematical misconceptions based on cognitive styles as reflective and impulsive, found that students with a reflective cognitive style are more prone to errors in translation, strategy, systematization, symbols, and mathematics in general. These fallacies are mostly caused by a lack of conceptual knowledge and mistakes in interpreting issues and recording solution procedures.

Although the findings are more relevant to youngsters under the age of twelve, Ginsberg (1977) offers numerous remarks concerning errors:

- Errors are the outcome of well-established methods and procedures. Faulty rules that support mistakes have logical roots.
- Too frequently, youngsters see arithmetic as apart from everyday issues. (As you shall see below, many algebra mistakes and erroneous thinking are linked to arithmetic misconceptions and faulty thinking) (for example, fractions).
- Children typically demonstrate a mismatch between formal and informal knowledge.

Siegler et al. (2013) focused on the errors students commonly make and misinterpret in the context of fractions. Similarly, Kieran (2007) addressed algebraic misconceptions, particularly those related to concepts of linearity. Boaler (2022) suggests overcoming misconceptions can be achieved through a developmental mindset. Based on her research, she argues that students who view mistakes as learning opportunities are more likely to overcome these issues and develop a deeper understanding of mathematics. Additionally, she advocates for the use of visual aids and the creation of interactive learning environments to help reduce misconceptions throughout this process.

According to Kabadaş & Yavuz Mumcu (2024) research, instructors mostly used traditional ways to diagnose and eliminate students' misunderstandings. In most cases, professors corrected students' faults superficially rather than fully focusing on their reasoning. Regarding approaches for correcting student misconceptions, teachers preferred to immediately advise students that their replies were incorrect rather than supporting students in recognizing their own faults. The findings highlight the need to increase teacher and student understanding of math misconceptions.

Various investigations have found that mathematical misconceptions generate major issues in other courses. One of them is identifying the elements contributing to variances in individual student performance in high school physics courses. Several studies show a positive association between students' mathematical ability and their test grades in high school physics. Erdoğan et al. (2014) aimed to assess the impact of mathematical misconception on student achievement in kinematics teaching. If long-term misconceptions are not detected, they might impede future mathematical learning. For example, Woodward & Howard (1994) demonstrated that a continuous, superficial knowledge of mathematics allows students to apply incorrect algorithms or repair processes, eventually leading to persistent and deep-seated misconceptions.

Finally, the literature reveals a consistent pattern: Students' errors are frequently the result of procedural knowledge rather than a thorough comprehension of the topics involved. The solution is to build relational understanding, use visual aids, and cultivate a mentality that accepts mistakes as part of the learning process.

RESEARCH METHODS

In this study, a mixed-methods approach was used to examine and analyze students' mathematical misconceptions. This method not only considered numerical patterns but also integrated contextual insights related to the mistakes made by students and the approaches teachers applied in class to correct them. By combining these perspectives, a more comprehensive and clear picture of the misconceptions and their remedies was presented.

For this research, we selected institutions that offer English-medium education across Iraq and are distinguished by their academic success. While we did not focus on student demographics, these reputable schools are composed of Kurdish, Turkmen, and Arab students and do not practice co-education. They have achieved significant success in Olympiads, including producing students who have won bronze medals in various years in the International Mathematics Olympiad—an unprecedented accomplishment in Iraq’s history (Delan Serhang & Bnar Karim, 2024).

In coordination with the school administrations, students from different classes, generally at the same academic level, participated in and supported our study. These schools are well-suited to Iraq’s demographic structure. We recognized that selecting fixed schools catering to only one specific student demographic, whether public or private, would not have provided a comprehensive understanding. It is also worth noting that these schools are selective in their admissions process. As private, tuition-based institutions, they are highly sought after by the public, yet they offer opportunities to students from all backgrounds, including free education for high achievers (Celik & Kara, 2024). Additionally, they are known for fostering a strong sense of camaraderie and mutual respect among both students and teachers.

Participants

Our study included 150 students from grades 9 to 12, equally distributed in terms of gender and academic level, from Stirling Schools in the cities of Kirkuk, Basra, and Baghdad, Iraq. The students and teachers participated in this study entirely voluntarily, and they were informed that they could leave the study at any time without any issues. Additionally, gender diversity was taken into account during the selection of students. Additionally, feedback from ten mathematics teachers working in these schools was analyzed to explore their perspectives on the causes of mathematical misconceptions and effective teaching approaches.

Ethical Procedures

The Stirling Schools Committee accepted this research with a decision dated April 4, 2024, and designated E-007, No:0037. Parental agreement was sought from students participating in the study, and confidentiality norms were followed within the scope of ethical guidelines.

Data Collection

A diagnostic math test was designed to identify misconceptions in three key areas: algebra, fractions, and geometry. This test, consisting of 30 questions, included 10 questions from each subject area, measuring both conceptual and procedural understanding. Algebra problems addressed errors in the distributive property, equation solving, and expression manipulation. Fraction questions assessed students' ability to add, subtract, multiply, and divide fractions. Focused on common mistakes included misunderstanding the connection between the numerator and denominator. Geometry problems involved calculating perimeter and area, which students often misunderstood.

Each student's response was examined to determine the sort of error made. Errors are classed as follows:

- Conceptual Errors: Misunderstanding of basic principles.
- Procedural Errors: Mistakes in following proper processes or methods.
- Application Errors: Misapplication of principles to real-world or difficult issues.

The following table 1 displays the percentage distribution of mistakes across the three mathematical disciplines.

Table 1. Diagnostic Test Error Distribution

Topic	Conceptual Errors (%)	Procedural Errors (%)	Application Errors (%)
Algebra	35	25	10
Fractions	40	30	15
Geometry	20	35	25

Teacher Interviews

In-depth semi-structured interviews were performed with ten math teachers. The interview questions aimed to understand teachers' perspectives on typical student misunderstandings.

- Teachers' perspectives on common student misconceptions.
- Instructional techniques for addressing these misconceptions.
- Challenges in correcting mistakes and misconceptions.

A thematic analysis of the interviews revealed common themes such as rote memorization, difficulties with abstract concepts, and the efficacy of visual aids and collaborative learning.

Classroom Observations

Five sessions (one in each school) were observed to collect real-time examples of student misunderstandings. Observations focused on how students approached problem-solving and how teachers handled mistakes. This offered additional qualitative data to better understand the environment in which misunderstandings emerged and how they were addressed.

RESULTS AND DISCUSSION

The findings from both quantitative and qualitative data indicated significant tendencies in the types and causes of mathematical misunderstandings. The examination of test results and teacher feedback provided insight into both student faults and effective solutions. The results of the diagnostic exam revealed that fractions had the greatest percentage of conceptual mistakes (40%), followed by algebra (35%), and geometry (20%). This shows that students struggle more with comprehending the ideas behind fractions, such as the link between numerators and denominators, as opposed to geometry, where their errors were primarily procedural.

Procedural mistakes accounted for the largest amount of errors in geometry (35%), indicating the difficulty students encounter when using geometric formulae accurately, especially when separating area and perimeter computations. On the other hand, the percentage of procedural mistakes in algebra was comparatively lower (25%), suggesting that although students may be aware of the techniques, their application of them is often incorrect because of inadequate conceptual understanding.

Table 2. Pre-and Post-Intervention Correct Responses

Topic	Pre-Test (%)	Post-Test (%)	Improvement (%)
Algebra	55	75	+20
Fractions	45	70	+25
Geometry	60	80	+20

After implementing several interventions, such as conceptual training, visual aids, and collaborative problem-solving, there was a noteworthy enhancement in the students' comprehension of all subjects. Students showed the biggest improvement in fractions, with a 25% increase in correct answers. This implies that emphasizing conceptual instruction—for example, by representing fractions using number lines or pie charts—was very successful.

The qualitative information gleaned from teacher interviews corresponded with the quantitative results in numerous recurring themes:

Memorization Reliance: Instructors have seen that a large number of students largely rely on memorization of steps without genuinely comprehending the ideas. This is especially troublesome regarding fractions, as students frequently use whole-number reasoning to solve fractions (e.g., adding the numerators and denominators directly).

Misapplication of Rules: Teachers said that students commonly apply rules acquired in one context to different issues. For example, several students mistakenly utilized the distributive property in algebraic formulas like $(b + c) = ab + c$, demonstrating a limited comprehension of algebraic laws.

Effective Visual Aids: Teachers found that using visual representations (e.g., area models, geometric diagrams, fraction bars) greatly increased student understanding. For example, in geometry, utilizing diagrams to physically express the difference between perimeter and area helped many students overcome conceptual difficulties. These findings support Boaler's (2022) advocate for visual and collaborative learning environments.

Classroom observations corroborated these findings, with students who used visual aids and worked in collaborative groups being more likely to detect and fix their misunderstandings. Teachers that encouraged students to convey their thoughts to peers reported a greater incidence of students acknowledging their own flaws. While improvements were shown in all domains following the intervention, the results indicated that students' procedural mistakes in geometry, while reduced, remained substantially greater than in other areas. This discovery may indicate possible challenges in properly comprehending geometric operations, even with improved conceptual knowledge. Similarly, qualitative results suggested difficulties in shifting from memory to deeper comprehension, which might help explain why some students continued to misapply rules after treatments.

How Can Misconceptions Be Removed

Misconceptions are misunderstandings or misinterpretations based on incorrect definitions. They are caused by 'naive theories,' which inhibit learners' rational reasoning. Misconceptions may take many shapes. According to Ojose (2015), misunderstandings "exist partly because of students' overriding need to make sense of the instruction they receive." The shift typically produces cognitive conflicts and dissonance for learners since it forces them to unlearn what they have previously acquired.

Mestre (1987) describes an excellent inductive approach for various applications. There are three steps for this: first, the identification and evaluation of qualitative knowledge; second, the search for and analysis of quantitative understanding; and third, the recognition and identification of conceptual thinking.

Furthermore, providing students with counter-examples to their beliefs is important. A self-discovered counterexample will have a substantially stronger and more enduring effect. When so engaged, erroneous beliefs might be partially relaxed. One of the most important tasks in resolving mathematics misconceptions is to alter students' mental framework. That is, just telling (e.g., lecturing) the learner about a mistake is only partially effective. Internally, the misperception must be addressed in part through the student's belief systems and cognition.

Teacher's Role In Spotting Misconceptions

We must work together to detect and exploit misconceptions that create cognitive conflict. Misconceptions should not be ignored but rather embraced as mathematical opportunities to expand and increase knowledge and comprehension. These can be investigated in conversation-rich environments such as concept cartoons, factual/false/sometimes claims, or odd-one-out balloon discussion games. Counterexamples are extremely beneficial during in-depth discussions. In dialogue-rich activities, confronting misunderstandings allows children to reevaluate their ideas, which helps to make their thinking more visible, leading to more complicated reasoning and reflection. It teaches students that misconceptions are usually the result of overgeneralizing from fundamental circumstances they have observed previously.

Alkramiti (2023) depicts a sequence of interactions between a teacher and students to identify and fix mathematical misconceptions. Figure 1 illustrates how a teacher may successfully identify students' misunderstandings by allowing them to give conceptual explanations and justifications for their mathematical solutions. Furthermore, changing these beliefs requires adopting proper teaching tactics.

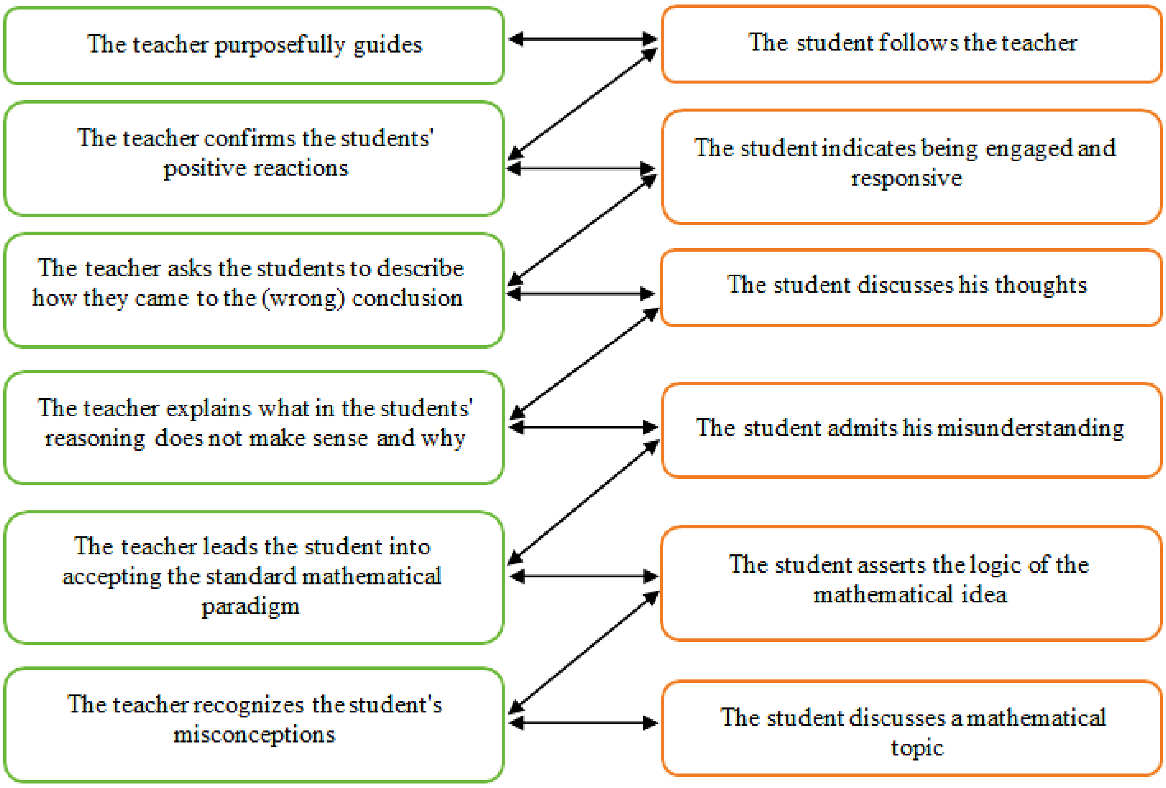

Figure 1. Interactions between teachers and students to discover and correct mathematical misconceptions

When a discussion focuses on misconceptions as a normal aspect of learning, students may feel more comfortable voicing their opinions. "The number one rule in my classroom is that I'm interested in how you got there, not where you are", Mackle (2016) states in his memoir, "and that it is okay to make mistakes." Please preserve a dictionary of math misunderstandings you face in your daily teaching, as this will be a helpful teaching resource, especially when shared with a professional learning network. Teachers have a critical role in identifying students' mathematical misconceptions.

According to Kleickmann et al. (2015), teachers may use a range of techniques to do this, depending on the nature of the students' misconceptions. Teachers will struggle to identify their students' misconceptions unless they are proficient in two critical areas: topic knowledge and pedagogical content knowledge.

As a result of this research, it is undeniable that attention must be drawn to several key points. First, it can be emphasized, especially to teachers and students, that a deeper understanding of important subjects like mathematics can be achieved through focusing on comprehension rather than memorization. Instead of students merely memorizing procedures, they should be guided to understand the logic behind each algebraic step.

Just as laboratories are frequently used in science classes, partial "mathematics laboratories" can be created. These can feature various visual aids, such as diagrams, number lines, and geometric models—common in everyday life—introduced while teaching relevant topics. This can help students more effectively learn complex concepts. When visual aids are used in class, students tend to better grasp mathematical relationships, making the knowledge more memorable.

Additionally, attention should be given to homework and assessments, a topic widely researched. When evaluating homework during or after lessons, instead of just marking answers as "right/wrong," providing comments that explain why a particular response is incorrect, or offering reference points (books, videos, websites) that encourage further investigation, can help students more effectively address their misconceptions.

Finally, motivating students and fostering a growth mindset is critically important. Mistakes are not failures but rather valuable learning opportunities. Let us not forget that mathematical misconceptions persist due to weaknesses in foundational understanding. Therefore, positive steps and interventions aimed at closing these gaps will lead to long-term improvements in student performance.

CONCLUSION

Mathematics, whether due to its nature or misconceptions attributed to it, has always been a source of misunderstanding for students. Teachers, who play a key role, must be aware of these mathematical misconceptions and ensure that they do not persist over long periods. These misconceptions pose significant barriers to student performance. Just as a preventive doctor administers early treatments to avoid more serious issues, teachers must take the necessary preemptive steps to reduce the need for "surgical interventions" later.

This approach will help nurture a society of healthier, happier students who enjoy mathematics as much as they do other arts like music and painting and who feel the satisfaction of solving problems correctly in their daily lives. This can be achieved through deliberate and targeted interventions designed to strengthen students' conceptual understanding.

This study identified the core areas where misconceptions are most common (algebra, fractions, and geometry) and demonstrated that treatments such as visual aids, peer collaboration, and growth mindset education can significantly reduce these errors. By shifting from a teacher-centered, procedural learning model to a more conceptually focused approach, educators can better equip students with the tools they need to overcome mathematical misconceptions.

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