



Analyzing students' epistemic actions in basic algebra using the Abstraction in Context (AiC) model: A descriptive qualitative case study

Alda Gemellia Munawwaroh^{1*}, Ikrar Pramudya², Siswanto³

^{1,2,3}Universitas Sebelas Maret, Indonesia

¹agmellia23@student.uns.ac.id, ²ikrarpramudya@staff.uns.ac.id, ³sis.mipa@staff.uns.ac.id

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ABSTRACT

Mathematical abstraction skills, which include concept recognition, relationship development, and new knowledge formation, are necessary for understanding basic algebra. The abstraction process can be analyzed through students' epistemic actions using the AiC Model, which helps identify how students recognize, develop, and actively construct mathematical concepts. This study aims to explore and describe the epistemic actions of seventh-grade students in performing mathematical abstraction, thereby providing insights that can be used to develop more effective algebra teaching strategies. The results show that only S1 was able to effectively apply epistemic actions in mathematical abstraction, even though this student had not yet reached the stage of constructing new knowledge. Meanwhile, other students experienced difficulties, especially in recognizing incorrect knowledge structures and performing constructing and building actions. These difficulties were largely due to incomplete algebraic knowledge, which prevented students from using theoretical thinking effectively and reorganizing their knowledge. Based on these findings, teachers are advised to emphasize a deep conceptual understanding of algebra rather than memorization. In addition, the use of contextual and visual representations should be strengthened to help students connect concrete and abstract concepts. Teachers can also design progressive tasks that guide students through recognition, relationship building, and systematic knowledge construction.



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INTRODUCTION

Hershkowitz, Schwarz and Dreyfus, (2001a) stated that abstraction is a chain of actions performed by individuals or groups and controlled by a specific motivation within a given context. Abstraction is a process of change that allows existing situations stored in memory to be combined with new situations generated from experience (Butuner and Ipek 2023). There are two aspects agreed upon by several researchers regarding abstraction, namely 1) a new mental object is produced as a result of the abstraction process, and 2) this new object separates certain features deemed irrelevant (general qualities) from other qualities (Hassan and Mitchelmore 2006). In other words, individuals require information they already know to solve problems and apply prior knowledge to the process of abstracting new knowledge (Memnun et al. 2019).

In mathematics, this process is called mathematical abstraction, in which students construct new mathematical structures from previously acquired mathematical structures (Kim and Hong 2016). According to the 1991 classification of mathematical subjects, a theory or structure can be considered abstract if it can be characterized by a certain degree of generalization and decontextualization (Ferrari 2003). Several concepts of abstraction used in approaches to mathematics learning include the formation of basic mathematical concepts (empirical abstraction), mathematical interpretation of realistic contexts (horizontal mathematics), and the development of mathematical theory (theoretical and vertical abstraction) (White and Mitchelmore 2010). In mathematics, algebra is a mathematical concept in the formal operational stage with arithmetic as its foundational knowledge. Arithmetic is a mathematical concept for the concrete operational stage. Thus, in algebra learning, there is a transition in students' thinking from concrete operational to formal operational (Mukhni, Mirna, and Khairani 2021).

In algebra, the process of abstraction can be integrated with the contextualization of problems. This is because algebra is closely related to the contextualization of problems in students' daily lives. Contextualization is a way for students to organize and conceptualize their views of a phenomenon and the implications of those views on their understanding, as well as their subsequent interaction with the phenomenon (Nilsson 2009). In other words, students inevitably have to use their empirical and theoretical knowledge simultaneously in the process of contextualizing algebraic problems, thereby obtaining the desired new knowledge structure.

The process of contextualization involves the conceptualization of students' understanding. The conceptual context is one of the specific contexts in mathematical understanding, which leads to a personal understanding of the learning situation. Conceptual understanding occurs when students create mental constructs, which are translated into two things: as a process and as an object. The development of concepts as objects involves developing concepts through knowledge structures. Meanwhile, the development of concepts as processes involves developing concepts based on interactions with the environment that can influence thinking (Reyes et al. 2019).

However, in practice, algebra teaching still pays little attention to conceptualization elements. A systematic literature review conducted by Pincheira and Alsina (2021) shows that conceptualization in Mathematical Knowledge for Teaching (MKT) in basic algebra material focuses on the development of functional thinking. Meanwhile, other elements that support algebraic thinking are overlooked, such as the use of concrete representations in the form of visualizations to analyze situations, analyze changes in various contexts, and very few apply and explore arithmetic generalizations. This is in line with the 2011 TIMSS results, which showed that Indonesia ranked 38th out of 42 countries in algebra scores. Indonesian students still struggle with procedural skills and conceptual understanding, and their understanding of structure is still limited to early algebra learning (Jupri, Sispiyati, and Chin 2021). This indicates that previous studies have primarily emphasized the outcomes of conceptual understanding rather than the epistemic processes underlying how students construct algebraic knowledge. The limited exploration of students' epistemic actions, such as recognizing, building-with, and constructing creates a gap in understanding the mechanisms of mathematical abstraction in algebra learning. Therefore, this study aims to address this gap by examining students' epistemic processes within the framework of the Abstraction in Context (AiC) model.

The process of mathematical abstraction is the process of generalizing problems from concrete objects to abstract forms, involving the implementation of empirical and theoretical knowledge (Mitchelmore and White 2007). In the RBC models perspective, the abstraction process begins with the need for a new structure and consists of three epistemic actions: recognizing, building-with, and constructing knowledge (Gürbüz and Ozdemir 2020). Recognizing involves empirical thinking, but that alone is not sufficient to form abstraction, which also requires theoretical thinking. The next action, building-with, refers to the use of available mathematical elements to achieve the goal. Finally, the action of constructing is more about assembling mathematical structures to produce new ones (Ozmantar and Monaghan 2007). This study will show how students' epistemic actions in solving basic algebra problems are analyzed based on the RBC theory framework.

RESEARCH METHODS

This study was the initial part of a series of didactical design research studies. The discussion and analysis in this study used a descriptive qualitative approach to analyze students' epistemic steps when performing mathematical abstraction before the metapedadidactic analysis stage. This study was a case study with the research subjects being seventh-grade students at a public junior high school in Grobogan Regency. The total number of DDR research subjects was 34, who participated in the initial ability and prospective analyses to gain an overview of their preliminary understanding of algebraic abstraction. From this group, three students were purposively selected for in-depth, test-based interviews. The purposive sampling technique was applied to ensure that the selected participants represented diverse levels of mathematical ability and distinct cognitive characteristics, allowing a more comprehensive exploration of epistemic variations in abstraction processes. The interview tasks were designed to assess mathematical abstraction based on the Abstraction in Context (AiC) model and consisted of three open-ended questions, each aimed at eliciting one or more epistemic actions,

recognizing, building-with, and constructing. Instrument validation was conducted in two stages. In the internal validation phase, expert reviewers within the research team examined the tasks to ensure conceptual clarity, content accuracy, and alignment with the AiC theoretical framework. In the external validation phase, independent mathematics educators and didactics experts evaluated the tasks to verify content relevance, cognitive demand, and contextual appropriateness for junior high school students. All interviews were audio-recorded and transcribed verbatim. The data were systematically analyzed by categorizing students' responses according to the epistemic actions of the AiC model and identifying emerging patterns in their abstraction processes. This multi-step procedure ensured methodological rigor, transparency, and the credibility of the qualitative findings.

RESULTS AND DISCUSSION

Abstraction is a verb that is theoretically defined as a series of actions performed by an individual or a group driven by specific motives within a particular context (Hershkowitz, Schwarz and Dreyfus, 2001a). In mathematics education, the process of abstract thinking is often referred to as mathematical abstraction. Mathematical abstraction enables the formation of new mathematical objects through the process of decontextualization from an object to a new object at a higher level (Ferrari 2003). This process involves several epistemic actions, namely the processes of constructing, recognizing, and building-with (Hershkowitz, Schwarz and Dreyfus, 2001a).

Recognizing mathematical structures occurs when students realize that these structures are inherent in certain mathematical situations. Recognizing can occur in two cases, namely 1) through analogy with other objects with similar structures that students have previously recognized, or 2) through specialization by realizing that these objects belong to a class in which all members have similar structures. Building-with is the effort to combine several existing structural parts to solve a problem or justify a statement. This process is likely to occur when students can utilize strategies, rules, or theorems to solve a given situation. Constructing is the main step in abstraction, where students assemble existing structural parts to produce a new structure. This process requires theoretical thinking and implies the reorganization of knowledge. The main characteristic of this action is the emergence of new knowledge (Dreyfus, Hershkowitz and Schwarz, 2001b).

These three epistemic actions do not occur sequentially like a chain but form a nest. In other words, the constructing action does not merely follow the recognizing and building-with actions linearly but simultaneously requires recognition and construction of the structures already built (Dreyfus, Hershkowitz and Schwarz, 2001b). This indicates that the three epistemic actions are interrelated but not a cause-and-effect relationship. In this study, we will analyze how the epistemic actions performed by the subjects on each question are analyzed. The three questions given to the subjects are presented below.

1. Question 1: Determine the results of the following algebraic operations! (Write down the steps to solve them.)
 - a. $2ab + 6ab - 21ab =$
 - b. $3p - 3q + 11p =$
 - c. $9m - 16n =$
2. Question 2: Create a single algebraic expression that has two different variables and one constant (it must include all of these elements). Then show which are the coefficients, variables, and constants of the algebraic expression!
3. Question 3: Two shopping packages have the following lists of items:
Package A: 15 storybooks, 10 picture books, and 10 colored pencils.
Package B: 3 boxes of storybooks, 4 bags of picture books, and 2 boxes of colored pencils.
How many items are in Package A? Give your reasoning!
How many items are in Package B? Give your reasoning!

The first question asks students to perform recognition and building with processes using their knowledge of number theory and algebraic operations. In the second question, students are expected to

perform the building-with process, which means that they can solve problems using their prior knowledge, such as knowledge about variables, coefficients, and constants.

In general, the third question asks students to perform the construction process using the provided table, recognize (the concept of variables), and build with (algebraic addition operations). The objective is to understand the concept of variables as quantities whose values are unknown.

Based on the results of 34 students' work, 16 students were able to solve problem number 1 well. Meanwhile, for problems number 2 and 3, only 7 students were able to show the correct answers. To find out how the epistemic action process occurred, interviews were conducted with three subjects selected through purposive sampling.

Among the three research subjects, only one student (S1) demonstrated the presence of mathematical abstraction, evidenced by the occurrence of complete epistemic action. The following is a quote from the interview with S1.

P : What about point b?

S1 : It is equal to 3p and 11p.

P : What makes them equal?

S1 : The p.

P : What is the result?

S1 : 14p, then subtract 3q.

P : Can it be calculated?

S1 : Yes, it becomes 11pq.

.....

P : From question no. 2, what was asked?

S1 : Finding the coefficients, variables, and constants, and giving one algebraic form.

P : Which are the coefficients, variables, and constants?

S1 : The constant is this, the variable is this, and the coefficient is this... (Fig. 2)

P : How many variables are there?

S1 : Two... but the letters are the same, so one variable.

.....

P : If you were asked to count the items in package A, how would you do it?

S1 : $15 + 10 + 10$ (Fig. 3)

P : Why that way?

S1 : I don't know, I just counted.

P : How about package B?

S1 : 3 boxes, 15 storybooks each, so 3×15 .

P : What if the number of storybooks is unknown?

S1 : $3 + 4 + 2...$

P : Is the total in Package A the same as Package B?

S1 : Different...

P : Why?

S1 : Um... I don't know...

The interview excerpts are derived from the qualitative data collected in this study, as presented in Tables 1, 2, and 3 below.

b.)
$$\begin{aligned} 3p - 3q + 11p &= \\ 3p + 11p - 3q &= \\ 14p - 3q &= \end{aligned}$$

Figure 1. Results of S1 work no. 1

Jawab:

$$\begin{aligned} 17x + 8x + 2 \\ 2 &= \text{Konstanta} \\ x &= \text{Variabel} \\ 17 - 8 &= \text{Koefisien} \end{aligned}$$

Figure 2. Results of S1 work no. 2

a. Berapa jumlah barang dalam Paket A? Berikan alasanmu!

Jawab:

$$\begin{aligned} \text{jumlah barang paket A} &= 15 + 10 + 10 \\ &= 25 + 10 \\ &= 35 \quad (35 \text{ barang}) \\ \text{Jadi barang dalam paket A adalah } &35 \text{ barang} \end{aligned}$$

b. Berapa jumlah barang dalam Paket B? Berikan alasanmu!

Jawab:

$$\begin{aligned} \text{jumlah barang paket B} &= (3 \times 15) + (4 \times 10) + (2 \times 10) \\ &= 45 + 40 + 20 \\ &= 85 + 20 \\ &= 105 \\ \text{Jadi barang dalam paket B adalah } &105 \text{ barang} \end{aligned}$$

Figure 3. Results of S1 work no. 3

Based on the interview excerpts, it is known that in question no. 1, S1 carried out the building-with process, namely S1 used knowledge about number theory and operations of similar or dissimilar terms in algebra. However, when prompted a little, the student wavered and gave the wrong answer. This shows that the structure of prior knowledge was not strong.

Then, in question no. 2, S1 showed a complete epistemic action. The building-with process is a process in which students try to create an algebraic form. In this case, students also involved a recognition process in which they recognized and realized that the structure used was correct. The construction process in this excerpt is implied when the student realizes that the structure used previously cannot solve the problem, and the student can directly replace it with the appropriate knowledge structure (..same.. means there is one variable..) after being guided a little by the researcher.

In question no. 3, S1 performed the abstraction process well. The construction process occurred when the student realized that the knowledge structure previously used to solve the problem could not achieve the expected goal, then replaced it with a new knowledge structure, resulting in a better answer. However, S1 had not yet reached a new conceptual conclusion about the variable (whose value was unknown).

Unlike S1, subject S2 did not demonstrate good mathematical abstraction processes. Based on the analysis results, it was found that errors or misconceptions in the initial knowledge structure significantly impacted the failure of the knowledge construction process by the student. The following is a quote from the interview with S2.

- P : Try explaining point a.
S2 : $2ab + 6ab$ are added together, because you have to add them first, then once you're done, 8 is subtracted from 21 ab, which equals 13 ab (Fig. 4).
P : Why is the result positive?
S2 : Because the debt is 8, is debt ...

- P : Okay, what did you do?
- S2 : $5+2-3y = 7-3y$ (Fig. 5)
- P : Which one is the constant?
- S2 : The two constants... I don't know, just thinking.
- P : What's the difference between a coefficient and a constant?
- S2 : Maybe the constant is the odd one... The 2 isn't odd, it's just a regular number.
- ...
- S2 : There are 10 colored pencils, but I don't know how many in each pack.
- P : How many in total?
- S2 : 10.
- ...
- P : Package B?
- S2 : 3 boxes of storybooks (unknown number inside), 2 boxes of colored pencils (unknown), 4 bags of drawing books.
- P : How many drawing books?
- S2 : One in each bag, so 4.

The interview excerpts are derived from the qualitative data collected in this study, as presented in Figures 4, 5, and 6 below.

a.) $2ab + 6ab - 21ab =$
 ~~$9ab + 6ab - 8ab = 7ab$~~
 $2ab + 6ab = 8ab$
 $8ab - 21ab = 13ab$

Figure 4. Results of S1 work no. 1

Jawab:
 ~~$5 + 2 - 3y$~~
 $(5) + 2 - (3y) = 7 - 3y$

Figure 5. Results of S1 work no. 2

a. Berapa jumlah barang dalam Paket A? Berikan alasanmu!

Jawab:
 15 buku cerita
 10 buku gambar
 10 pensil warna → variabel
 karena kita tidak tau isi pensil warna tersebut

b. Berapa jumlah barang dalam Paket B? Berikan alasanmu!

Jawab:
 3 kardus buku cerita → variabel
 4 kantong buku gambar → variabel
 2 box pensil warna
 karena kita tidak tau isi dalam kardus itu
 karena kita tidak tau isi pensil warna tersebut

Figure 6. Results of S1 work no. 3

Based on the results of the work and interviews with S2, it is known that the subject is not yet able to apply mathematical abstraction processes properly. In question number 1 during the recognition process, S2 did not achieve the correct knowledge structure. "The debt is 8, then 21 is the money paid like that." As a result, the building-with process also produced an incorrect knowledge structure.

In question number 2, it is evident that S2 does not have the correct basic knowledge structure related to algebraic elements. The subject cannot recognize and identify algebraic elements. This indicates that S2 has never constructed their knowledge about algebraic elements but only recalls it during the learning process. In question number 3, S2 does not even apply epistemic steps. S2 chose to follow their imagination to obtain information, which was then interpreted as knowledge.

Subject S3 demonstrated different abilities in each problem given. In questions 1 and 3, the abstraction process occurred well. However, in question 2, the abstraction process resulted in the construction of incorrect knowledge due to errors in the basic knowledge structure possessed by S3. The following is an excerpt from an interview with S3.

- P : Explain part a.
S3 : $2ab + 6ab - 21ab = 8ab - 21ab = -13...$
P : Why negative?
S3 : Because 8 minus 21... eight cannot be subtracted from twenty-one.
.....
P : Which coefficient is it?
S3 : The three.
P : Which are the variables?
S3 : $3a$ and $6b...$
P : So a and b are constants?
S3 : Yes, a and b are constants... 5 is a coefficient because it's a number.
.....
P : Are the numbers in package A and B the same or different?
S3 : Different, because A can be counted (35 items) but B cannot.

The interview excerpts are derived from the qualitative data collected in this study, as presented in Figures 7, 8, and 9 bellow.

a.) $2ab + 6ab - 21ab = 2ab + 6ab$
 $= 8ab$
 $= 8ab - 21ab$
 $= -13ab$

Figure 7. Results of S3 work no.1

Jawab:
 $3a + 4b + 5$
 koefisien = ~~ada~~ $3, 4$
 variabel = $3a + 4b$
 konstanta = 5

Figure 8. Results of S3 work no.2

a. Berapa jumlah barang dalam Paket A? Berikan alasanmu!
 Jawab:
 $\text{Paket A} = 15 \text{ bc} + 10 \text{ bg} + 10 \text{ pw} = 35$
 karena paket A bersikan 15 buku cerita, 10 buku gambar, 10 pensil warna

 b. Berapa jumlah barang dalam Paket B? Berikan alasanmu!
 Jawab:
 $\text{Paket B} = 3 \text{ kardus buku cerita} + 10 \text{ kg} + 2 \text{ bpw}$
 karena paket B bersikan 3 kardus buku cerita, 10 kantong buku gambar, 2 box pensil warna

Figure 9. Results of S3 work no.3

Based on the results of the work and S3 interviews, it is known that the subject correctly applied epistemic steps in question number 1. The subject was able to recognize his knowledge so that it could be used in the building-with process, namely when performing algebraic calculations. In question number 2, the subject showed inconsistency in his answers. The subject kept changing their answers during the interview. This was due to the subject's lack of a proper knowledge structure regarding algebraic terms.

In question number 3, the subject was able to apply epistemic steps well when faced with contextual problems. The student was able to recognize their knowledge of the concepts of variables, coefficients, and constants. The student was also able to realize that in point b, the variables used were different, so they could not be added together. The building-with process went well, but the subject stopped before being able to write down the requested algebraic form. However, when guided during the interview process, the subject was able to provide one algebraic form for the problem, although the use of variables was still not entirely accurate. Subject S3 was also able to achieve the process of constructing new knowledge, where the subject finally understood that the difference between points a and b lies in the variable as a quantity whose value is known (point a) and a variable whose value is unknown (point b).

This ability is significantly different from when solving problem number 2, where the subject was unable to create a single example of an algebraic form and identify its elements. This indicates an incomplete knowledge structure, necessitating efforts to reconstruct knowledge for S3.

Table 1. Comparison of Students' Epistemic Actions in the Abstraction Process in Basic Algebra Learning

Aspect / Question	S1	S2	S3
Question 1 (Building-with Recognition)	Applied building-with process using number theory and operations on similar/dissimilar terms; wavered with slight guidance; prior knowledge structure weak	Failed recognition; knowledge structure incorrect ("The debt is 8, then 21 is the money paid like that"); building-with process produced incorrect knowledge	Correct recognition and building-with process for algebraic calculations; prior knowledge sufficient
Question 2 (Recognition Building-with Construction)	Complete epistemic action; recognized correct knowledge; construction implied with guidance ("same" means one variable)	Did not recognize algebraic elements; building-with process incorrect; never constructed knowledge	Inconsistent; changed answers during interview; basic knowledge of algebraic terms insufficient; construction failed
Question 3 (Abstraction Construction)	Applied abstraction process; constructed knowledge by replacing incorrect structure; did not reach new conceptual conclusion about unknown variable	Did not apply epistemic steps; relied on imagination; no abstraction	Applied epistemic steps well in contextual problem; recognized concepts of variables, coefficients, constants; building-with good; construction achieved; distinguished known vs unknown variable; slight inaccuracy in final algebraic form
Overall Strengths	Able to apply epistemic actions with guidance; partial knowledge construction	Very limited epistemic application; lacks basic knowledge structure	Strongest in abstraction and construction with guidance; inconsistent in some tasks due to incomplete knowledge structure
Overall Weaknesses	Cannot construct entirely new knowledge independently	Knowledge structure incomplete; fails to construct or apply abstraction	Inconsistent across problems; requires knowledge reconstruction in some cases

The analysis of students' epistemic actions in basic algebra reveals a clear variation in the ability to perform mathematical abstraction, reflecting differences in prior knowledge and cognitive readiness. Subject S1 demonstrated relatively strong abstraction skills, particularly in questions 2 and 3, where the student effectively engaged in recognizing, building-with, and partially constructing new knowledge. The student's ability to adjust previously applied knowledge structures after guidance indicates an emerging metacognitive awareness and aligns with the cognitive frameworks of the Abstraction in Context (AiC) model, which emphasizes the interconnectedness of recognition, building-with, and

construction in the abstraction process (Hershkowitz et al., 2001). Despite this, S1 was not yet able to achieve full knowledge construction regarding variables whose values were unknown, which is consistent with prior studies showing that even capable students may require scaffolding to reach higher levels of abstraction (Mitchelmore & White, 2007; Sezgin Memnun et al., 2017).

In contrast, S2 struggled significantly across all questions. The inability to correctly recognize and apply algebraic elements in the building-with process illustrates that the student lacked a robust knowledge structure, relying instead on memorization or imagination rather than meaningful construction of knowledge. This finding is consistent with research on the role of prior knowledge in abstraction, which indicates that misconceptions or incomplete understanding of algebraic concepts impede the ability to perform higher-order cognitive actions such as knowledge construction (Ozmantar & Monaghan, 2007; Greenes, 1995). S2's failure to engage epistemic steps highlights the importance of systematic instruction that explicitly targets the development of foundational knowledge structures, supporting students to internalize and apply algebraic concepts rather than merely recalling procedures.

Subject S3 exhibited variable performance, demonstrating successful epistemic actions in problems 1 and 3 but struggling in problem 2 due to gaps in the foundational knowledge of algebraic elements. The student's ability to eventually construct new knowledge in question 3, differentiating between known and unknown variables, reflects the effectiveness of guided scaffolding and situational prompts in facilitating conceptual understanding (Ferrari, 2003; Kim & Hong, 2016). This aligns with the RBC+C theoretical perspective, which emphasizes the iterative interplay between recognition, building-with, and constructing to achieve meaningful abstraction (Sezgin Memnun & Aydin, 2017). Overall, these findings reinforce that successful mathematical abstraction depends not only on procedural proficiency but also on the availability of coherent, well-structured prior knowledge and the ability to engage in metacognitive regulation during problem-solving.

These findings also resonate with previous large-scale assessments and literature reviews. Jupri, Sispiyati, and Chin (2021) reported that Indonesian students continue to experience difficulties in both procedural and conceptual aspects of algebra, with limited understanding of algebraic structures beyond early algebraic reasoning. The varied performance observed among S1, S2, and S3 in this study reflects these broader national trends, suggesting that weak conceptual foundations hinder students' ability to engage in epistemic processes such as recognizing and constructing knowledge. Furthermore, the findings extend the work of Pincheira and Alsina (2021), who found that research on mathematical knowledge for teaching (MKT) in basic algebra has largely emphasized functional thinking. By focusing on students' epistemic actions, recognizing, building-with, and constructing, this study contributes to a deeper understanding of how conceptualization and abstraction actually occur in classroom contexts, addressing a gap previously identified in the literature.

Thus, students' ability to perform epistemic actions in basic algebra is strongly influenced by the strength and completeness of their prior knowledge structures. S1 demonstrated good abstraction skills, successfully engaging in recognition and building-with processes and approaching constructing, while S2 struggled due to weak prior knowledge, relying mainly on memorization or imagination. S3 showed varied performance, able to construct new knowledge after guidance on certain problems. These findings highlight the importance of developing foundational knowledge structures and providing learning experiences that support metacognitive regulation to facilitate effective mathematical abstraction, in line with the Abstraction in Context (AiC) model and RBC+C theory.

CONCLUSION

Based on the results and explanation of students' epistemic action in learning basic algebra, it is known that only S1 is able to apply epistemic action well in the process of mathematical abstraction, although it has not been able to achieve the process of constructing new knowledge. While some other students still have difficulty in performing the mathematical abstraction process. Some of the problems found are, in the recognition process, where students are unable to realize that the knowledge structure used is not able to solve the problem given.

In the building-with process, students perform the building-with process using the wrong knowledge structures. The knowledge structures in question are: a) distinguishing and identifying

algebraic elements, b) mathematical terms or language of algebraic elements, c) concepts of variables, coefficients, and constants in algebra, d) concepts of similar and dissimilar terms. As well as the constructing process, where: a) students do not have a good knowledge structure so that students cannot use their theoretical thinking and b) students are also not accustomed to the process of reorganizing knowledge.

Many things can be behind this failure, including the absence of the correct basic knowledge structure to be used in the abstraction process, students also never construct the correct knowledge about the elements of algebra, but only memorize it in learning. In addition, the incomplete knowledge structure also causes students to be unable to solve different types of problems, even though the same knowledge is needed.

Based on these findings, mathematics teachers are encouraged to design instruction that actively supports students' epistemic actions in mathematical abstraction. This can be achieved by providing tasks that explicitly guide students through the stages of recognizing, building-with, and constructing knowledge, as well as offering opportunities to reorganize and apply algebraic concepts in varied contexts. Teachers should emphasize conceptual understanding over rote memorization and incorporate collaborative problem-solving to help students reflect on and refine their knowledge structures. For future research, studies could explore the effectiveness of instructional interventions that target specific epistemic actions across diverse student populations, investigate the longitudinal development of abstraction skills, and examine how digital tools or adaptive learning environments can support students' epistemic engagement in algebra and other areas of mathematics.

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